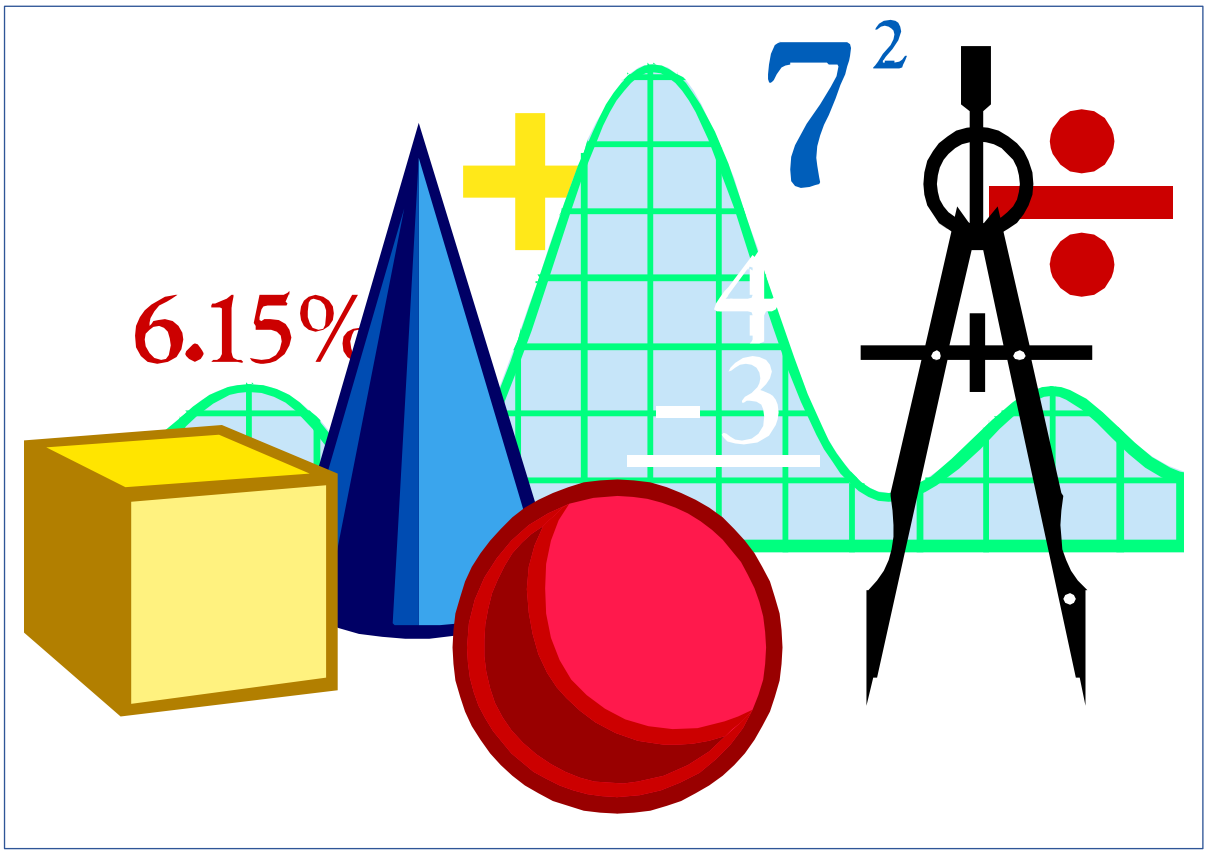
Year 11 > 12 Bridging Work
Summer Term 2023



Subject	**Maths and Further Maths**
Course	**A Level**
Awarding Body	**Edexcel**

Contents:

Course/specification overview

You will study combination of Pure mathematics, Mechanics and Statistics, with two thirds of the course focusing on the Pure. The aims and objectives of this qualification are to enable students to:

- understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems that require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively and recognise when their use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

<u>The topics you will be covering include:</u>

Pure: Proof and algebraic methods, Trigonometry, Coordinate geometry, The Binomial Expansion, Vectors, Integration, Differentiation, Exponentials and logarithms, Numerical methods, Functions, Sequences and Parametric Equations.

Mechanics: Kinematics, Forces, Dynamics, Statics, Moments, Variable acceleration and Vectors.
Statistics: Data Collection, Measuring and Representing data, Correlation, Hypothesis Testing, Linear regressions, Conditional Probability, Probability Distributions, The Binomial Distribution and The Normal Distribution.

Final Assessments:

You will sit 3 papers at the end of year 13. Each paper will be 2 hours and will make up one third of your final grade

- Paper 1 and Paper 2 will be based on Pure Mathematics
- Paper 3 will be on Applied Mathematics – Mechanics and Statistic

You will be allowed to use a calculator in all three papers

Our Department expectations

Maths A level is very hard work. I mean it! It will take a lot of effort from you to reach your potential in this subject. However, a good qualification in this subject is valued very highly by Universities and employers. It can also be very rewarding and enjoyable. A minimum grade of a 7 is required for this course. However, this is no guarantee that you will do well on the course! (The same is true of an 8/9)

We will need to push on at quite a pace to get through the work. You will have homework set regularly during the week which will need to be completed before the next lesson. There will be a lot of work to complete. You will need to get yourself organised quickly. There will be regular tests on work covered. There will be an initial skills test in September to ensure that you have mastered essential algebra skills.

The staff in Hayes Maths Department love maths and are keen to help. We will always endeavour to find time for anyone who comes to see us with problems. Our staff and year 13 prefects run a support session on Mondays and Wednesdays.

You will need to purchase your own textbooks and calculators. You may wish to do this sooner rather than later so can you get familiar with your calculator and start looking ahead at the content.

Calculators:

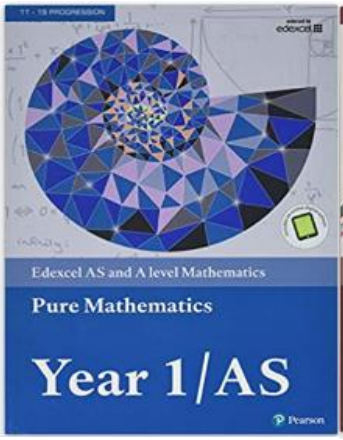
You will need a calculator which stores statistical distribution tables (you current GCSE calculator probably does not do this). We recommend the Casio FX-991EX. However, if you are taking Further Maths or are planning on studying a Maths related degree, you may wish to purchase a graphical calculator which stores the statistical distribution tables. We recommend the CASIO FX-CG50

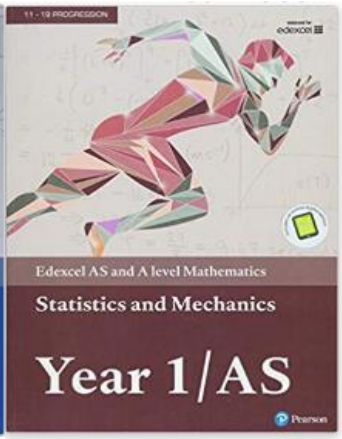
Casio FX-991EX

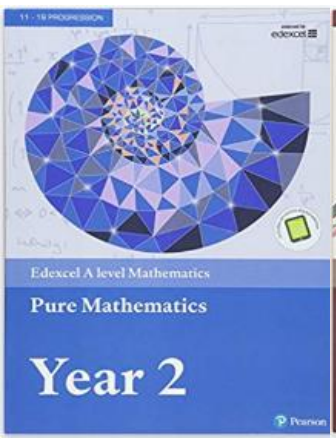
CASIO FX-CG50

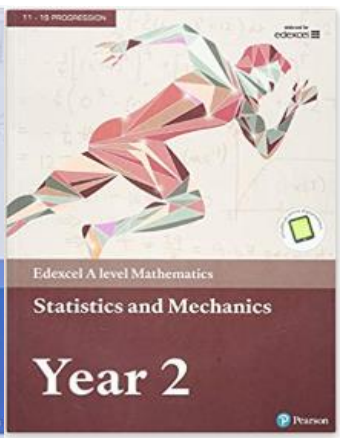
Textbooks:

You will need to purchase these books. You could try looking on ebay or other sites for second hand copies if you want to save some money. They are published by Pearson.









Review/revise

It is **vitally important** that you spend some time working through the questions in this booklet - you will need to have a good knowledge of these topics **before** you commence your course in September. You should have met all the topics before at GCSE. These skills should be secure and automatic as they form the foundation on which the rest of the course will be built.

Work through the introduction to each chapter, making sure that you understand the examples. Then tackle the exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly. The answers are given at the back of the booklet. It is a requirement of the course that this preparatory work is completed.

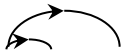
We will test you in September to check how well you understand these topics, so it is essential that you have worked through the booklet before then. If you do not pass this test, you will be placed on a support contract and will be expected to attend support sessions and complete extra work until you have achieved the required standard. You will then be re-tested in October. A mock test is provided at the back of this booklet. **You must bring your answers to this AND evidence of completion of each of the preceding exercises to your first lesson in September**. If you fail to complete this work your continuation on the course will be reconsidered.

CONTENTS

Chapter 1: REMOVING BRACKETS

To remove a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

Examples



1) $3(x + 2y)$ $= 3x + 6y$

2) $-2(2x - 3)$ $= (-2)(2x) + (-2)(-3)$

$= -4x + 6$

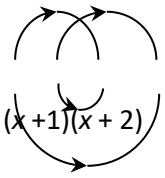
To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

Examples:

1) $(x + 1)(x + 2)$ $= x(x + 2) + 1(x + 2)$

or



$(x + 1)(x + 2)$ $= x^2 + 2 + 2x + x$

$= x^2 + 3x + 2$

or

	x	1
x	x^2	x
2	$2x$	2

$(x + 1)(x + 2)$ $= x^2 + 2x + x + 2$

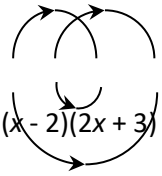
$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$

$= 2x^2 + 3x - 4x - 6$

$= 2x^2 - x - 6$

or



$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$

$= 2x^2 - x - 6$

EXERCISE A Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a - 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x - (x + 4)$
6. $5(2x - 1) - (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x - 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$

12. $(3 + 5x)(4 - x)$

Two Special Cases

Perfect Square:	**Difference of two squares:**
$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$	$(x - a)(x + a) \quad = x^2 - a^2$
$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$	$(x - 3)(x + 3) \quad = x^2 - 3^2$
	$= x^2 - 9$

EXERCISE B Multiply out

1. $(x - 1)^2$

2. $(3x + 5)^2$

3. $(7x - 2)^2$

4. $(x + 2)(x - 2)$

5. $(3x + 1)(3x - 1)$

6. $(5y - 3)(5y + 3)$

Chapter 2: LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain any x^2 or x^3 terms.

More help on solving equations can be obtained by downloading the leaflet available at this website: http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simplelinear.pdf

Example 1: Solve the equation $64 - 3x = 25$

Solution: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the x term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 - 2x$.

Solution:

Step 1: Begin by adding $2x$ to both sides $8x + 7 = 5$

(to ensure that the x terms are together on the same side)

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -¼$

Exercise A: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$

2) $5x - 2 = 13$

3) $11 - 4x = 5$

4) $5 - 7x = -9$

5) $11 + 3x = 8 - 2x$

6) $7x + 2 = 4x - 5$

Example 3: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$

(taking care of the negative signs)

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

Exercise B: Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\frac{y}{2} + 5 = 11$

Solution:

Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

Example 5: Solve the equation $\frac{1}{3}(2x+1) = 5$

Solution:

Step 1: Multiply by 3 (to remove the fraction) $2x + 1 = 15$

Step 2: Subtract 1 from each side $2x = 14$

Step 3: Divide by 2 $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

Example 6: Solve the equation $\frac{x+1}{4}+\frac{x+2}{5}=2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$

Step 3: Simplify the left hand side:

$$\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

Example 7: Solve the equation $x+\frac{x-2}{4}=2-\frac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x+\frac{12(x-2)}{4}=24-\frac{12(3-5x)}{6}$$

Simplify $12x+3(x-2)=24-2(3-5x)$

Expand brackets $12x+3x-6=24-6+10x$

Simplify $15x-6=18+10x$

Subtract 10x $5x-6=18$

Add 6 $5x = 24$

Divide by 5	$x = 4.8$

Exercise C: Solve these equations

1) $\frac{1}{2}(x+3)=5$

2) $\frac{2x}{3}-1=\frac{x}{3}+4$

3) $\frac{y}{4}+3=5-\frac{y}{3}$

4) $\frac{x-2}{7}=2+\frac{3-x}{14}$

Exercise C (continued)

5) $\frac{7x-1}{2}=13-x$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

FORMING EQUATIONS

Example 8: Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.

Therefore $n + (n + 1) + (n + 2) = 96$

$3n + 3 = 96$

$3n = 93$

$n = 31$

So the numbers are 31, 32 and 33.

Exercise D:

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.

Form an equation, letting n be the number of photographs one girl had at the **beginning**.

Hence find how many each has **now**.

Chapter 3: SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is $3x + 2y = 8$ ○

$5x + y = 11$ ○

In these equations, x and y stand for two numbers. We can solve these equations in order to find the values of x and y by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate y. We do this by making the coefficients of y the same in both equations. This can be achieved by multiplying equation ○ by 2, so that both equations contain $2y$:

$3x + 2y = 8$ ○

$10x + 2y = 22$ $2\times$○ = ○

To eliminate the y terms, we subtract equation ○ from equation ○. We get: $7x = 14$

i.e. $x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into ○:

$10 + y = 11$

$y = 1$

Therefore the solution is $x = 2$, $y = 1$.

Remember: You can <u>check</u> your solutions by substituting both x and y into the original equations.

Example: Solve $2x + 5y = 16$ ○

$3x - 4y = 1$ ○

Solution: We begin by getting the same number of x or y appearing in both equation. We can get $20y$ in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ ○

$15x - 20y = 5$ ○

As the SIGNS in front of 20y are DIFFERENT, we can eliminate the y terms from the equations by ADDING:

$23x = 69$ ❍+❍

i.e. $x = 3$

Substituting this into equation ❍ gives:

$6 + 5y = 16$

$5y = 10$

So… $y = 2$

The solution is $x = 3$, $y = 2$.

If you need **more help** on solving simultaneous equations, you can download a booklet from the following website:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-simultaneous1.pdf

Exercise:

Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$

$3x + 2y = 9$

2) $x + 3y = 0$

$3x + 2y = -7$

3) $3x - 2y = 4$

$2x + 3y = -6$

4) $9x - 2y = 25$

$4x - 5y = 7$

5) $4a + 3b = 22$

$5a - 4b = 43$

6) $3p + 3q = 15$

$2p + 5q = 14$

Chapter 4: FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1: Factorise $12x - 30$

Solution: 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

Example 2: Factorise $6x^2 - 2xy$

Solution: 2 is a common factor to both 6 and 2. Both terms also contain an x.

So we factorise by taking $2x$ outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

Example 3: Factorise $9x^3y^2 - 18x^2y$

Solution: 9 is a common factor to both 9 and 18.

The highest power of x that is present in both expressions is x^2.

There is also a y present in both parts.

So we factorise by taking $9x^2y$ outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

Example 4: Factorise $3x(2x - 1) - 4(2x - 1)$

Solution: There is a common bracket as a factor.

So we factorise by taking $(2x - 1)$ out as a factor.

The expression factorises to $(2x - 1)(3x - 4)$

Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$

The method is:

Step 1: Form two brackets (*x* …)(*x* …)

Step 2: Find two numbers that multiply to give *c* and add to make *b*. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.

Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$

The method is:

Step 1: Find two numbers that multiply together to make *ac* and add to make *b*.

Step 2: Split up the *bx* term using the numbers found in step 1.

Step 3: Factorise the front and back pair of expressions as fully as possible.

Step 4: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make 6 × -12 = -72 and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = 6x^2 - 8x + 9x - 12$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

$16x^2 - 25 = (2x)^2 - 5^2 = (2x + 5)(2x - 5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x + 4)(x - 4)$

and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x + 4y)(x - 4y)$

Factorising by pairing

We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can download a booklet from this website
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf

Exercise B

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$ (factorise by taking out a common factor)

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2y$

14) $8(x+1)^2 - 2(x+1) - 10$

Chapter 5: CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

Example 1: Make *x* the subject of the formula $y = 4x + 3$.

Solution: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with *x* the subject.

Example 2: Make *x* the subject of $y = 2 - 5x$

Solution: Notice that in this formula the *x* term is negative.

$y = 2 - 5x$

Add $5x$ to both sides $y + 5x = 2$ (the *x* term is now positive)

Subtract *y* from both sides $5x = 2 - y$

Divide both sides by 5 $x = \frac{2-y}{5}$

Example 3: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make *F* the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9 $9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C + 160}{5}$.

Exercise A

Make *x* the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \frac{x + 5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x - 5)}{9}$

Rearranging equations involving squares and square roots

Example 4: Make *x* the subject of $x^2 + y^2 = w^2$

Solution: $x^2 + y^2 = w^2$

Subtract y^2 from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving *x*)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

Example 5: Make *a* the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\frac{5a}{h}}$

Square both sides $16t^2 = \frac{5a}{h}$

Multiply by *h*: $16t^2h = 5a$

Divide by 5: $\frac{16t^2h}{5} = a$

Exercise B:

Make *t* the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

Example 6: Make *t* the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add *xt* to both sides: $a = b + yt + xt$

Now put the terms without a *t* on the left hand side:

Subtract *b* from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\dfrac{a-b}{y+x} = t$

So the required equation is $t = \dfrac{a-b}{y+x}$

Example 7: Make *W* the subject of the formula $T - W = \dfrac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \dfrac{2bT}{a+2b}$

If you need more help you can download an information booklet on rearranging equations from the following website:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-formulae2-tom.pdf

Exercise C

Make *x* the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \dfrac{2x + 3}{5x - 2}$

4) $\dfrac{x}{a} = 1 + \dfrac{x}{b}$

Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:

* factorising

* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x - 1)(x - 2) = 0$

Either $(x - 1) = 0$ or $(x - 2) = 0$

So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x - 2) = 0$

Either $x = 0$ or $(x - 2) = 0$

So $x = 0$ or $x = 2$

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$

We can then tell that $a = 2$, $b = 3$ and $c = -12$.

Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, there is an information booklet downloadable from this web site: http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-quadraticequations.pdf

EXERCISE

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$ b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

Chapter 7: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of *y*.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$

2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$

3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the *a*'s)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the *c*'s)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the *d* terms i.e. by subtracting the powers)

Exercise A

Simplify the following:

1) $b \times 5b^5$ = (Remember that $b = b^1$)

2) $3c^2 \times 2c^5$ =

3) $b^2c \times bc^3$ =

4) $2n^6 \times (-6n^2)$ =

5) $8n^8 \div 2n^3$ =

6) $d^{11} \div d^9$ =

7) $\left(a^3\right)^2$ =

8) $\left(-d^4\right)^3$ =

More complex powers

Zero index:

Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number *a*.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$ (you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers:

Fractional powers correspond to roots: $a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$$8^{1/3} = \sqrt[3]{8} = 2 \qquad 25^{1/2} = \sqrt{25} = 5 \qquad 10000^{1/4} = \sqrt[4]{10000} = 10$$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Exercise B:

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) 5^{-2}

5) 18^{0}

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^3 \times x^{-2}$

15) $\left(x^2 y^4\right)^{1/2}$

Practice Booklet Test

This is the review test that you must complete and bring full worked solutions to your first lesson in September. A similar skills test will be given at in early September. Students should be aware that if they underperform in this text the will be required to attend additional afterschool sessions until they have mastered all of the fundatmental skills which are covered in this booklet.

You may NOT use a calculator

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Expand and simplify

   (a) $(2x + 3)(2x - 1)$ (b) $(a + 3)^2$ (c) $4x(3x - 2) - x(2x + 5)$

2. Factorise

   (a) $x^2 - 7x$ (b) $y^2 - 64$ (c) $2x^2 + 5x - 3$ (d) $6t^2 - 13t + 5$

3. Simplify

   (a) $\dfrac{4x^3y}{8x^2y^3}$ (b) $\dfrac{3x+2}{3} + \dfrac{4x-1}{6}$

4. Solve the following equations

   (a) $\dfrac{h-1}{4} + \dfrac{3h}{5} = 4$ (b) $x^2 - 8x = 0$ (c) $p^2 + 4p = 12$

5. Write each of the following as single powers of x and / y

   (a) $\dfrac{1}{x^4}$ (b) $(x^2y)^3$ (c) $\dfrac{x^5}{x^{-2}}$

6. Work out the values of the following, giving your answers as fractions

(a) 4^{-2} (b) 10^0 (c) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

7. Solve the simultaneous equations $3x - 5y = -11$

$5x - 2y = 7$

8. Rearrange the following equations to make x the subject

(a) $v^2 = u^2 + 2ax$ (b) $V = \frac{1}{3}\pi x^2 h$ (c) $y = \frac{x+2}{x+1}$

9. Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form

SOLUTIONS TO THE EXERCISES

CHAPTER 1:

Ex A

1) $28x + 35$ 2) $-15x + 21$ 3) $-7a + 4$ 4) $6y + 3y^2$ 5) $-4x - 4$

6) $7x - 1$ 7) $x^2 + 5x + 6$ 8) $t^2 - 7t + 10$ 9) $6x^2 + xy - 12y^2$

10) $4x^2 + 4x - 24$ 11) $4y^2 - 1$ 12) $12 + 17x - 5x^2$

Ex B

1) $x^2 - 2x + 1$ 2) $9x^2 + 30x + 25$ 3) $49x^2 - 28x + 4$ 4) $x^2 - 4$

5) $9x^2 - 1$ 6) $25y^2 - 9$

CHAPTER 2

Ex A

1) 7 2) 3 3) 1½ 4) 2 5) -3/5 6) -7/3

Ex B

1) 2.4 2) 5 3) 1 4) ½

Ex C

1) 7 2) 15 3) 24/7 4) 35/3 5) 3 6) 2 7) 9/5 8) 5

Ex D

1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

CHAPTER 3

1) $x = 1, y = 3$ 2) $x = -3, y = 1$ 3) $x = 0, y = -2$ 4) $x = 3, y = 1$

5) $a = 7, b = -2$ 6) $p = 11/3, q = 4/3$

CHAPTER 4

Ex A

1) $x(3 + y)$ 2) $2x(2x – y)$ 3) $pq(q – p)$ 4) $3q(p – 3q)$ 5) $2x^2(x - 3)$ 6) $4a^3b^2(2a^2 – 3b^2)$

7) $(y – 1)(5y + 3)$

Ex B

1) $(x – 3)(x + 2)$ 2) $(x + 8)(x – 2)$ 3) $(2x + 1)(x + 2)$ 4) $x(2x – 3)$ 5) $(3x -1)(x + 2)$

6) $(2y + 3)(y + 7)$ 7) $(7y – 3)(y – 1)$ 8) $5(2x – 3)(x + 2)$ 9) $(2x + 5)(2x – 5)$ 10) $(x – 3)(x – y)$

11) $4(x – 2)(x – 1)$ 12) $(4m – 9n)(4m + 9n)$ 13) $y(2y – 3a)(2y + 3a)$ 14) $2(4x + 5)(x – 4)$

CHAPTER 5

Ex A

1) $x = \frac{y+1}{7}$ 2) $x = 4y - 5$ 3) $x = 3(4y + 2)$ 4) $x = \frac{9y+20}{12}$

Ex B

1) $t = \frac{32rP}{w}$ 2) $t = \pm\sqrt{\frac{32rP}{w}}$ 3) $t = \pm\sqrt{\frac{3V}{\pi h}}$ 4) $t = \frac{P^2 g}{2}$ 5) $t = v - \frac{Pag}{w}$ 6) $t = \pm\sqrt{\frac{r-a}{b}}$

Ex C

1) $x = \frac{c-3}{a-b}$ 2) $x = \frac{3a+2k}{k-3}$ 3) $x = \frac{2y+3}{5y-2}$ 4) $x = \frac{ab}{b-a}$

CHAPTER 6

1) a) -1, -2 b) -1, 4 c) -5, 3 2) a) 0, -3 b) 0, 4 c) 2, -2

3) a) -1/2, 4/3 b) 0.5, 2.5 4) a) -5.30, -1.70 b) 1.07, -0.699 c) -1.20, 1.45

d) no solutions e) no solutions f) no solutions

CHAPTER 7

Ex A

1) $5b^6$ 2) $6c^7$ 3) b^3c^4 4) $-12n^8$ 5) $4n^5$ 6) d^2 7) a^6 8) $-d^{12}$

Ex B

1) 2 2) 3 3) 1/3 4) 1/25 5) 1 6) 1/7 7) 9 8) 9/4 9) ¼ 10) 0.2 11) 4/9 12) 64

13) $6a^3$ 14) x 15) xy^2

Watch

If you would like extra help with any of the GCSE topics in the material above you may want to visit https://corbettmaths.com/contents/

There are also some highly recommended tranisiton form GCSE to A level video here: https://amsp.org.uk/resource/gcse-alevel-transition-resources

If you are looking for something to push you beyond these skills you may want to work through:
https://www.drfrostmaths.com/sow.php?year=A%20Level%202017&term=Pure%201

Listen to

You may be interested in the following podcasts:

The Numberphile Podcast - Interviews with people who love numbers and mathematics. Hosted by Brady Haran, maker of the Numberphile series on YouTube.

The Secrets of Mathematics - A series of talks and lectures from Oxford Mathematicians exploring the power and beauty of their subject. These talks would appeal to anyone interested in mathematics and its ever-growing range of applications from medicine to economics and beyond.

Women in Math: The Limit Does Not Exist - This podcast is an effort to promote visibility of women in mathematics. Inspired by the fact that women are vast minority in higher mathematics, Women in Math: The Limit Does Not Exist serves to increase enrolment and participation of women in mathematics and STEM courses

Girls Talk Math - Girls Talk Math is a free math day camp for high school girls in the Research Triangle area (NC) hosted by the Mathematics Department of the University of North Carolina at Chapel Hill. Campers complete challenging problem sets in areas of mathematics that go beyond the high school curriculum, research the lives of female mathematicians, and share what they learned through blog posts and podcasts. Starting in 2018, Girls Talk Math has a sister chapter at the University of Maryland.

Math Ed Podcast - Interviews with mathematics education researchers about recent studies. Hosted by Samuel Otten, University of Missouri. www.mathedpodcast.com Produced by Fibre Studios

Breaking Math Podcast - Breaking Math is a podcast that aims to make math accessible to everyone, and make it enjoyable. Every other week, topics such as chaos theory, forbidden formulas, and more will be covered in detail. If you have 45 or so minutes to spare, you're almost guaranteed to learn something new!

Read

You may wish to purchase the course textbooks (see section 2) and start through the examples.

Additionally, you may find the following books interesting, especially if you are considering studying Mathematics beyond A level:

Does God Play Dice by *Ian Stewart*
Chaos by *James Gleick*
The Codebook by *Simon Singh*
The Mathematics of Ciphers by *S.C. Coutinho*
In Code by *Sara Flannery*
A Brief History of Time by *Stephen Hawking*
Thinking About Mathematics by *Stewart Shapiro*
A Mathematician's Apology by *G. H. Hardy*
Fermat's Last Theorem by *Simon Singh*
Journey Through Genius: The Great Theorems of Mathematics by *William Dunham*
The Equation That Couldn't Be Solved by *Mario Livio*
Kepler's Conjecture by *George Szpiro*
The Music of the Primes by *Marcus du Sautoy*
Four Colors Suffice by *Robin Wilson*

Engaging in the content of some of these books will be particularly useful when you come to complete you UCAS applications.

Research

The following website offers interactive introductory activities for each of the chapters in A level Maths. You may wish to start looking at these to give yourself a head start for September:

https://amsp.org.uk/about/integral

Username: amsp-HayesSchool190
Password: ExponentialDegree188!

Complete

For your first lesson in September you MUST have completed the material above. Your teacher will expect you to BRING your answers to the assessment on pages 43 and 44 to the first lesson. One of your first activities will be to mark this as a class.

You may also want to get into the habit of completing the Higher Plus Corbett Maths 5-day challenge each day:
https://corbettmaths.com/5-a-day/gcse/

Maths is like anything, if you don't use it you lose it. Doing just these five questions each day and then marking them will help you to keep your skills up.

Further Maths

At Hayes, Further Maths students complete all of their regular Maths A level during their Maths and Further Maths Lessons in year 12. Once this is complete we move onto the Further Maths content. Students complete the exams for both A levels at the end of Year 13.

This means that there is not much extra you can do to prepare for Further Maths than you would do for Maths. However, it is worth bearing in mind that the pace will be twice as fast. Being absolutely solid on the skills mentioned is essential and we would highly recommend engaging with Corbett Maths 5-a-day and looking ahead at the integral Maths website described in section 7.

Appendices/Resources – what else?

There are a range of additional useful resources for students self-studying AS/A level Mathematics and Further Mathematic described here
https://amsp.org.uk/resource/self-studying-a-level-maths

You may wish to download and print off the following formula booklet as you will use this constantly throughout the course

https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/Pearson_Edexcel_A_Level_GCE_in_Mathematics_Formulae_Book.pdf